Distributed slack bus model formulation for the Holomorphic Embedding Load flow Method (HELM)

Openmod Lightening Talk

Via Zoom

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Power Flow

$\|V\|$	Voltage magnitude	P	Active power injection
θ	Voltage phase angle	Q	Reactive power injection

Variables	Slack Bus	PQ Buses	PV Buses (Generators)
Known	$\|V\|\ ;\ \theta$	$P\ ;\ Q$	$P\ ;\ \|V\|$
Unknown	$P\ ;\ Q$	$\|V\|\ ;\ \theta$	$Q\ ;\ \theta$

Total active power load is known,
but the power losses are unknown.

Distributed slack bus model

This model states that the total power losses must be distributed amongst a group of buses and not just the slack bus.

Distribution based on participation factor:

$$\sum_{i=1}^{N} F_i = 1$$

$$F_i = \begin{cases} \dfrac{P_{g\,i}}{\sum_{k\in e} P_{g\,k}}, & if\ i \in e \\ 0, & if\ i \notin e \end{cases}$$

$e \rightarrow$ Set of PV buses and the slack bus.

$P_{g\,i} \rightarrow$ Scheduled active power generation at bus i.

$$P_{g\,slack} = \sum_{i=1}^{N} P_{d\,i} - \sum_{\substack{i=1 \\ i\neq slack}}^{N} P_{g\,i}$$

$Ploss \rightarrow$ New unknown variable

HELM

Bus	Original equation	Embedded equation
Slack	$V_i = V_i^{sp} \angle 0°$	$V_i(s) = 1 + \left(V_i^{sp} - 1\right) s$
PQ	$\sum_{k=1}^{N} Y_{ik}\, V_k = \frac{{S_i}^*}{{V_i}^*}$	$\sum_{k=1}^{N} Y_{ik\ series}\, V_k(s) = s\, \frac{{S_i}^*}{{V_i}^*(s^*)} - s\, Y_{i\ shunt}\, V_i(s)$
PV	$\sum_{k=1}^{N} Y_{ik}\, V_k = \frac{{S_i}^*}{{V_i}^*}$	$\sum_{k=1}^{N} Y_{ik\ series}\, V_k(s) = \frac{s\, P_i - j\, Q_i(s)}{{V_i}^*(s^*)} - s\, Y_{i\ shunt}\, V_i(s)$
	$\lvert V_i \rvert = V_i^{sp}$	$V_i(s) * {V_i}^*(s^*) = 1 + \left(\left(V_i^{sp}\right)^2 - 1\right) s$

HELM

$$P_i \rightarrow P_i + F_i \, Ploss$$

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Slack	$V_i = V_i^{sp} \angle 0°$	$V_i(s) = 1 + \left(V_i^{sp} - 1\right) s$
PQ	$\sum_{k=1}^{N} Y_{ik} V_k = \frac{S_i^*}{V_i^*}$	$\sum_{k=1}^{N} Y_{ik\,series} V_k(s) = s \frac{S_i^*}{V_i^*(s^*)} - s\, Y_{i\,shunt} V_i(s)$
PV	$\sum_{k=1}^{N} Y_{ik} V_k = \frac{S_i^*}{V_i^*}$	$\sum_{k=1}^{N} Y_{ik\,series} V_k(s) = \frac{s\, P_i - j\, Q_i(s)}{V_i^*(s^*)} - s\, Y_{i\,shunt} V_i(s)$
	$\lvert V_i \rvert = V_i^{sp}$	$V_i(s) * V_i^*(s^*) = 1 + \left(\left(V_i^{sp}\right)^2 - 1\right) s$

New PV equation preserving the algebraic nature of the problem:

$$\sum_{k=1}^{N} Y_{ik\,trans} V_k(s) = \frac{s\,P_i + F_i\,Ploss(s) - j\,Q_i(s)}{V_i^{\,*}(s^*)} - s\,Y_{i\,shunt}\,V_i(s)$$

Power series expansion: $V_i(s) = \sum_{n=0}^{\infty} V_i[n]\,s^n = V_i[0] + V_i[1]\,s + V_i[2]\,s^2 + \cdots$

Equation to calculate the n^{th} term of the power series:

$$\sum_{k=1}^{N} Y_{ik\,trans} V_k[n] - F_i\,Ploss[n] + j\,Q_i[n]$$

$$= P_i\,W_i^{\,*}[n-1] + F_i\left(\sum_{x=1}^{n-1} Ploss[x]\,W_i^{\,*}[n-x]\right) - j\left(\sum_{x=1}^{n-1} Q_i[x]\,W_i^{\,*}[n-x]\right) - Y_{i\,shunt}\,V_i[n-1]$$

New PV equation preserving the algebraic nature of the problem:

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Unbalance the equation system

- New unknown variable: $Ploss$
- New constant: $P_{g\ slack}$

Three bus system:

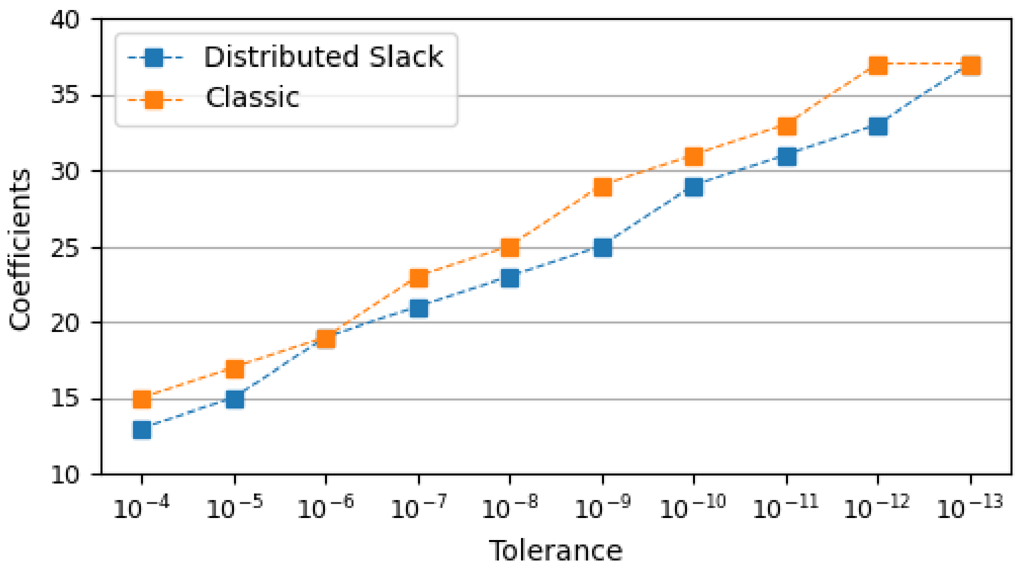
1. Slack bus
2. PQ bus
3. PV bus

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_{21} & -B_{21} & G_{22} & -B_{22} & 0 & -B_{23} & 0 & 0 \\ B_{21} & G_{21} & B_{22} & G_{22} & 0 & G_{23} & 0 & 0 \\ G_{31} & -B_{31} & G_{32} & -B_{32} & 0 & -B_{33} & -F_3 & 0 \\ B_{31} & G_{31} & B_{32} & G_{32} & 1 & G_{33} & 0 & 0 \\ G_{slack1} & -B_{slack1} & G_{slack2} & -B_{slack2} & 0 & -B_{slack3} & -F_{slack} & 0 \\ B_{slack1} & G_{slack1} & B_{slack2} & G_{slack2} & 0 & G_{slack3} & 0 & 1 \end{bmatrix}}_{8 \times 8} \begin{bmatrix} V_{1\,re}[n] \\ V_{1\,im}[n] \\ V_{2\,re}[n] \\ V_{2\,im}[n] \\ Q_3[n] \\ V_{3\,im}[n] \\ Ploss[n] \\ Q_{slack}[n] \end{bmatrix} = [known]$$

case1354pegase grid

Model	Losses (MW)
Classic slack bus	1672.14
Distributed slack bus	1660.83

Losses calculated by the classic and distributed slack bus model.

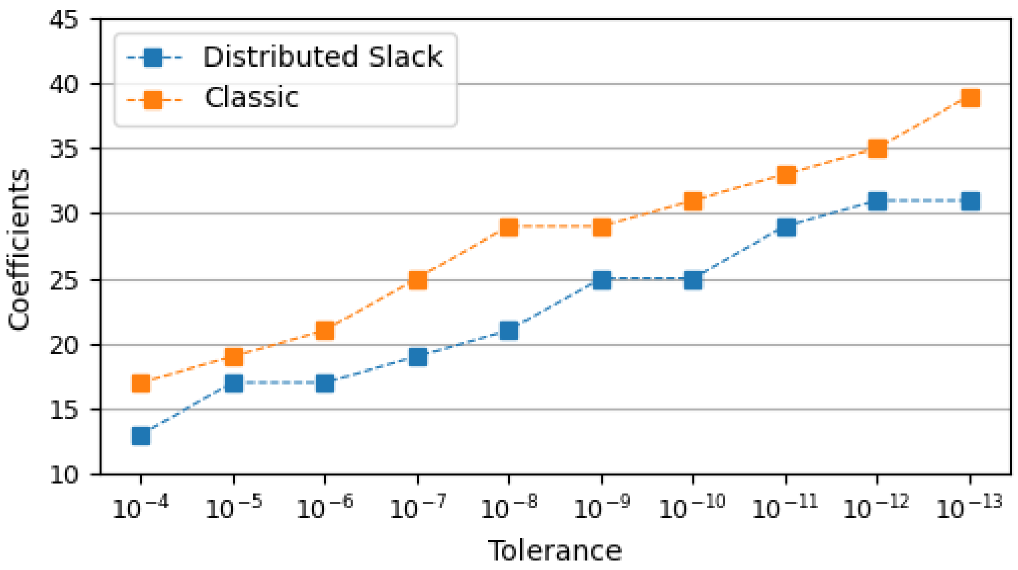


Coefficients calculated by classic and distributed slack bus HELM vs. tolerance

case2869pegase grid

Model	Losses (MW)
Classic slack bus	2802.73
Distributed slack bus	2755.98

Losses calculated by the classic and distributed slack bus model.



Coefficients calculated by classic and distributed slack bus HELM vs. tolerance

References

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